Honors Pre-Calculus Summer Work

This packet is to help you review various topics that are considered to be prerequisite knowledge upon entering Honors Pre-Calculus. It is due on the 1st day of the semester!!! Your first grade of the semester will include information from this packet following up with a summer work quiz

- Show all of your work for credit! Questions with NO work will receive NO credit!!
- All questions will be graded for correctness.
- Box your answers!
- NO CALCULATOR UNLESS OTHERWISE STATED!

1 - Parent Functions – Know how to graph each of the following parent functions, including any x and y intercepts and the domain and range.

<table>
<thead>
<tr>
<th></th>
<th>A) Linear  ( y = x )</th>
<th>B) Quadratic  ( y = x^2 )</th>
<th>C) Cubic  ( y = x^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:</td>
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<td>R:</td>
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D) Square Root  \( y = \sqrt{x} \)  

E) Cube Root  \( y = \sqrt[3]{x} \)  

D:                      
R:                      

2- Transformation-

Directions - Graph each of the following. If you don’t remember, use your graphing calculator to help you determine the patterns. But you need to be able to do these graphs without your calculator!

A.  \( y = \sqrt{x-2} - 3 \)  

B.  \( y = (x+2)^2 + 1 \)  

C.  \( y = |x| - 1 \)  

D.  \( y = \sqrt[3]{x+1} - 2 \)  

E.  \( y = (x-3)^3 + 2 \)  

F.  \( y = x + 3 \)
3- **Graphing Quadratics** - To graph a quadratic equation in standard form, \( y = ax^2 + bx + c \), find the important points of the graph by following the steps:

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
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<tbody>
<tr>
<td><strong>Y-intercept:</strong></td>
<td>If a point is the y-intercept of the curve, then that is the point at which the graph crosses the y-axis. Since this point is on the y-axis, then the x-coordinate must be 0. Substitute zero in for x and solve for y.</td>
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<tr>
<td><strong>Vertex:</strong></td>
<td>x-coordinate of the vertex: ( x = -\frac{b}{2a} ). y-coordinate of the vertex: substitute the value found for the x-coordinate into the original equation and solve for y.</td>
</tr>
<tr>
<td><strong>X-intercepts:</strong></td>
<td>If a point is an x-intercept of the curve, then it is a point at which the graph crosses the x-axis. Since these points are on the x-axis, then the y-coordinates must be 0. Substitute zero in for y and solve for x by factoring or using the quadratic formula.</td>
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</table>

*No calculator, but you should also be able to graph with the use of your calculator.*

**A. Directions** – Given \( y = -3x^2 - 6x + 2 \), find and graph.

a. y-intercept  
b. vertex  
c. x-intercepts

4- **Domain and Range**

- **Domain of a function** \( f(x) \): the set of all real numbers variable \( x \) can take such that the expression defining the function is real. The domain can also be given explicitly. Values not in the domain are those that yield division by zero or that yield a negative under an radical with even root.

  Ex1) \( f(x) = \frac{2}{x - 3} \). Domain: \( \{x|x \neq 3\} \) or \( (-\infty,3),(3,\infty) \)

  Ex2) \( f(x) = \sqrt{x} \). Domain: \( \{x|x \geq 0\} \) or \( [0,\infty) \)

  Ex3) \( f(x) = \{(1,2),(2,-3),(5,2),(6,7)\} \). Domain: \( \{x|x = 1,2,5,6\} \)

- **Range of a function** \( f(x) \): the set of all y values that the function takes when \( x \) takes values in the domain. (This is more easily determined from the graph)

  Ex1) \( f(x) = \frac{2}{x - 3} \). Range: \( \{y|y \neq 0\} \) or \( (-\infty,0),(0,\infty) \)

  Ex2) \( f(x) = \sqrt{x} \). Range: \( \{y|y \geq 0\} \) or \( [0,\infty) \)

  Ex3) \( f(x) = \{(1,2),(2,-3),(5,2),(6,7)\} \). Range: \( \{y|y = -3,2,7\} \)

**Directions:** Find the Domain of the following functions

a. \( f(x) = x^2 + 4 \)  
b. \( f(x) = \frac{1}{x} + \frac{2}{x + 2} \)  
c. \( g(x) = \frac{x}{x^2 - 5x} \)  
d. \( h(x) = \sqrt{4 - x} \)  
e. \( g(x) = \sqrt{x^2 + 1} \)
5- Exponents
Directions - Simplify using only positive exponents and no calculator!!!

Properties:

\[
\begin{array}{ccc}
\text{Properties:} & a^n \cdot a^m = a^{n+m} & (a^n)^m = a^{mn} & a^{-n} = \frac{1}{a^n} \\
& a^0 = 1, a \neq 0 & \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} & \frac{a^m}{a^n} = a^{m-n} \\
& a^{-n} = \frac{1}{a^n} & \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m & \frac{a^m}{c^{-m}} = \frac{a^m}{b^m} \\
& a^{-n} = \frac{1}{a^n} & (a^{-m})^{-n} = a^n & \frac{a^m}{b^n} = \frac{a^{m-n}}{b^n}
\end{array}
\]

A. \(\left(\frac{81}{64}\right)^{\frac{1}{3}}\)
B. \((27^{-2})^{\frac{1}{3}}\)

C. \(\frac{(3x^2)^{-1}}{6x^{-3}}\)
D. a. \(-2^4\),   b. \((-2)^4\)

E. \(\frac{3^{-5} \cdot 3^{-10}}{3^2}\)

F. \((4^{-1} + 2^{-1})^2\)

G. a. \((13y)^{-1}\),   b. \(13y^{-1}\)
H. \(8^{-1} \cdot 8\)
6- **Factoring**—Factor each completely

- Look for a common factor
- Difference of two squares: \( a^2 - b^2 = (a - b)(a + b) \)
- Perfect square trinomial: \( a^2 \pm 2ab + b^2 = (a \pm b)^2 \)
- Factorable trinomial (Target Sum/Target Product)
  - Guess and Check
  - Grouping
  - Sum/Difference of Cubes
    - \( a^3 + b^3 = (a + b)(a^2 - ab + b^2) \)
    - \( a^3 - b^3 = (a - b)(a^2 + ab + b^2) \)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Answer</th>
</tr>
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<tbody>
<tr>
<td>( 4x^2 + 27x + 35 )</td>
<td>( -28y^2 + 7t^2 )</td>
</tr>
<tr>
<td>( 4x^3 + 8x^2 - 5x - 10 )</td>
<td>( 8x^3 - 27 )</td>
</tr>
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</table>

7 - **Solving Quadratic Equations**

Since the following are equations, we can now go a step further and solve for \( x \) by factoring or using the quadratic formula.

<table>
<thead>
<tr>
<th>Expression</th>
<th>( -3x^2 - 5x + 12 = 0 )</th>
<th>( 12 - 5x^2 = 8x )</th>
<th>( 4y - 2 = y^2 )</th>
<th>( 225-b^2 = 0 )</th>
<th>( x^2 = x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 4y^2 = 25 )</td>
<td>( (y - 3)^2 = 10 )</td>
<td>( 3x^2 + 2x - 1 = 0 )</td>
<td>( 3x^2 + 3x^2 - 27x - 27 = 0 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8. Rational Expressions

A- Hint: factor and cancel!

\[
\begin{align*}
\text{a.} & \quad \frac{a^3 - a^2 - 6a}{a^2 - 9} \quad & \quad \text{b.} & \quad \frac{a^3 - 2a^2 + a - 2}{2 - a} \\
\text{c.} & \quad \frac{z^3 + 8}{z^2 - 2z + 4} \quad & \quad \text{d.} & \quad \frac{4ab^2}{ab^2 + a} \\
\text{e.} & \quad \frac{x^2 + 6x + 8}{x^2 - 4}
\end{align*}
\]

B - Perform the operation and write the result in reduced form.

\[
\begin{align*}
\text{a.} & \quad \frac{a - 1}{a} \cdot \frac{a^2}{a^2 - 1} \quad & \quad \text{b.} & \quad \frac{a^2 - 9}{a + 2} \cdot \frac{a^2 + 4a + 4}{a^2 - a - 6} \\
\text{c.} & \quad \frac{x + 4}{x^2} + \frac{x^2 - 16}{x}
\end{align*}
\]

C- Simplify: Hint: get a common denominator!

\[
\begin{align*}
\text{a.} & \quad \frac{2}{2x+1} - \frac{5}{(2x+1)^2} \quad & \quad \text{b.} & \quad 3 - \frac{4}{x+2} \\
\text{c.} & \quad \frac{1}{x^2 - 3x + 2} + \frac{2}{x^2 - 4} \\
\text{d.} & \quad \frac{3}{x - 2} + \frac{5}{2 - x} \\
\text{e.} & \quad \frac{y}{y^2 + 4y + 4} + \frac{3}{y^2 + y - 2}
\end{align*}
\]

D- Hint: get a common denominator in the numerator and multiply by the reciprocal, or multiply by LCD/LCD.

\[
\begin{align*}
\text{a.} & \quad \frac{1 - \frac{y}{3}}{3 - y} \quad & \quad \text{b.} & \quad \frac{1}{\frac{y+1}{y}} - \frac{\frac{1}{y}}{\frac{y+1}{y}} \\
\text{c.} & \quad \frac{x}{x-1} + \frac{1}{x + 2}
\end{align*}
\]
9- Radicals-

Directions – Simplify each of the radicals without a calculator.

a. \( \sqrt{128} \)  
b. \( \sqrt{9a^3b^7} \)  
c. \( \sqrt[4]{24a^4b^3} \)  
d. \( \frac{75}{\sqrt[4]{a^6}} \)  
e. \( \frac{5}{\sqrt{7}} \)  
f. \( \frac{3y}{4-\sqrt{7}} \)

10- Graphing and finding Linear Equations

A. Write an equation of the line with x-intercept of 3 and y-intercept of 5.

b. Write an equation of a line that goes through (2, 1) and (-2, 3).