Algebra Review: Linear Equations

Example: \[ 4x - 2(1 - x) = -38 \]
\[ 4x - 2 + 2x = -38 \]
\[ 6x - 2 = -38 \]
\[ 6x = -36 \]
\[ x = -6 \]

Solve each equation:

1) \[ 2p + 5 = 13 \]

2) \[ 12 + 2b = 2 + 5b \]

3) \[ 4x + 5 + 5x + 40 = 180 \]

4) \[ 2(4x + 4) = x + 1 \]

5) \[ 2(x + 5) = 3(x - 2) \]

6) \[ 180 - x = 3(90 - x) \]

7) \[ 3(180 - y) = 2(90 - y) \]

8) \[ 6x - 3(6 - 5x) + 3x = 10 - 4(2 - x) \]

9) \[ \frac{1}{2}(6 + 4x) - \frac{1}{4}(8x - 12) = \frac{1}{2}(2x - 4) \]

10) \[ 5x - [7 - (2x - 1)] = 3(x - 5) + 4(x + 3) \]
Algebra Review: Proportions

Definition: \( \frac{a}{b} = \frac{c}{d} \) if and only if \( ad = bc \)

Examples:

1) \( \frac{3}{2} = \frac{y}{22} \)
   
   \( 3(22) = 2y \)
   
   \( 66 = 2y \)
   
   \( 33 = y \)

2) \( \frac{x+4}{5} = \frac{x-2}{3} \)

   \( 3(x+4) = 5(x-2) \)

   \( 3x + 12 = 5x - 10 \)

   \( 22 = 2x \)

   \( 11 = x \)

Solve the following proportions using the format in the examples:

1) \( \frac{7}{2} = \frac{y}{3} \)

2) \( \frac{7}{3} = \frac{21}{x} \)

3) \( \frac{25}{15} = \frac{10}{x} \)

4) \( \frac{10}{6x+7} = \frac{6}{2x+9} \)

5) \( \frac{4}{x-3} = \frac{6}{x+3} \)

6) \( \frac{3x-5}{2} = \frac{x-15}{4} \)

7) \( \frac{2-4x}{-6} = \frac{6x-8}{10} \)

8) \( \frac{x+2}{5} = \frac{4}{x+1} \)

9) \( \frac{2}{x-3} = \frac{x-2}{6} \)
Algebra Review: Systems of Equations

Substitution Method

Example: \( y = 5 - 2x \)  
Solution: Substitute \( 5 - 2x \) for \( y \) in equation 2

\[
5x - 6y = 21 \\
5x - 6(5 - 2x) = 21 \\
5x - 30 + 12x = 21 \\
17x - 30 = 21 \\
17x = 51 \\
x = 3 \quad \text{(don't forget to find y)}
\]

Solve each system of equations by the substitution method.

1) \( y = 2x + 5 \)  
   \( 3x - y = 4 \)  

4) \( 8x + 3y = 26 \)  
   \( 2x = y - 4 \)  

2) \( x = 8 + 3y \)  
   \( 2x - 5y = 8 \)  

5) \( x - 7y = 13 \)  
   \( 3x - 5y = 23 \)  

3) \( 3x + 2y = 71 \)  
   \( y = 4 + 2x \)  

6) \( 3x + y = 19 \)  
   \( 2x - 5y = -10 \)
Elimination Method

Example:  \(3x + 4y = -10\)  
\[\begin{align*}
5x - 2y &= 18 \quad \times 2 \\
10x - 4y &= 36 \\
13x &= 26 \\
X &= 2
\end{align*}\]

Now substitute 2 for \(x\) and solve for \(y\) 
\[\begin{align*}
3(2) + 4y &= -10 \\
4y &= -16 \\
Y &= -4
\end{align*}\]

Example:  \(5x - 2y = -19\)  
\[\begin{align*}
5x - 2y &= 18 \quad \times 3 \\
15x - 6y &= -57 \\
2(-3) + 3y &= 0 \\
4x + 6y &= 0 \\
19x &= -57 \\
X &= -3
\end{align*}\]

Solve each system of equations using the elimination method. Use the format shown in the examples.

1)  \(3x + 4y = 9\)  
\[-3x - 2y = -3\]

4)  \(4x - 6y = -26\)  
\[-2x + 3y = 13\]

2)  \(5x + 3y = 30\)  
\[3x + 3y = 18\]

5)  \(2x - 8y = 24\)  
\[3x + 5y = 2\]

3)  \(3x + y = -3\)  
\[X + 4y = 10\]

6)  \(5x - 9y = 47\)  
\[6x + 2y = 18\]
Algebra Review: The Coordinate Plane

Name the coordinates of each point:

1) M \hspace{1cm} 6) T

2) N \hspace{1cm} 7) U

3) K \hspace{1cm} 8) V

4) R \hspace{1cm} 9) W

5) S \hspace{1cm} 10) Q

11) Name all the points shown that lie on the x-axis.

12) Name all the points shown on the y-axis.

13) What is the x-coordinate of every point that lies on the vertical line through P?

14) Which of the following points lie on a horizontal line through W?

\begin{align*}
(-2,1) & \quad (2,3) & \quad (1,-3) & \quad (-2,0) & \quad (0,-3) & \quad (2,0)
\end{align*}

15) Quadrant I \hspace{1cm} 16) Quadrant II \hspace{1cm} 17) Quadrant III \hspace{1cm} 18) Quadrant IV

Name all the points shown that lie in the quadrant indicated. (A point on an axis is not in any quadrant.)

Plot each point on the graph above.

19) A (2,1) \hspace{1cm} 20) B (5,0) \hspace{1cm} 21) C (0,3) \hspace{1cm} 22) D (-3,1)

23) E (-2,-1) \hspace{1cm} 24) F (1,-2) \hspace{1cm} 25) G (4,-2) \hspace{1cm} 26) H (-4,-3)
Algebra Review: Fractions

Examples:

a) \( \frac{8w}{2} \)

b) \( \frac{5x-10}{15} \)

c) \( \frac{x+6}{36-x^2} \)

\( 4w \)

\( \frac{5(x-2)}{15} \)

\( \frac{x+6}{(6-x)(6+x)} \)

\( \frac{x-2}{3} \)

\( \frac{1}{6-x} \)

1) \( \frac{14}{70} \)

2) \( \frac{75}{15} \)

3) \( \frac{18a}{36} \)

4) \( \frac{3x}{x} \)

5) \( \frac{x}{3x} \)

6) \( \frac{5bc}{10b^2} \)

7) \( \frac{-8y^3}{2y} \)

8) \( \frac{-18r^3t}{12rt} \)

9) \( \frac{3ab^2}{6bc} \)

10) \( \frac{6a+12}{6} \)

11) \( \frac{9x-6y}{3} \)

12) \( \frac{33ab-22b}{11b} \)

13) \( \frac{x+2}{3x+6} \)

14) \( \frac{2c-2d}{2c+2d} \)

15) \( \frac{t^2-1}{t-1} \)

16) \( \frac{5a+5b}{a^2-b^2} \)

17) \( \frac{b^2-25}{b^2-12b+35} \)

18) \( \frac{a^2+8a+16}{a^2-16} \)

19) \( \frac{3x^2-6x-24}{3x^2+2x-8} \)
Algebra Review: Radical Expressions

Examples:

a) \( \sqrt{56} \)  

b) \( \sqrt[3]{7} \)  

c) \( (3\sqrt{7})^2 \)

Solutions:

a) \( \sqrt{56} = \sqrt{4 \cdot 14} = 2\sqrt{14} \)

b) \( \sqrt[3]{7} \cdot \sqrt[3]{3} = \frac{\sqrt[3]{7}}{\sqrt[3]{3}} = \frac{\sqrt[3]{21}}{\sqrt[3]{3}} = \frac{\sqrt[3]{21}}{\sqrt[3]{3}} \)

c) \( (3\sqrt{7})^2 = (3\sqrt{7})(3\sqrt{7}) = 3 \cdot 3 \cdot \sqrt{7} \cdot \sqrt{7} = 9 \cdot \sqrt{49} = 9 \cdot 7 = 63 \)

Simplifying the following:

1) \( \sqrt{36} \)  

2) \( \sqrt{81} \)  

3) \( \sqrt{24} \)  

4) \( \sqrt{98} \)  

5) \( \sqrt{300} \)  

6) \( \sqrt[4]{1} \)  

7) \( \sqrt[5]{5} \)  

8) \( \sqrt[25]{80} \)  

9) \( \sqrt[12]{2 \sqrt{3}} \)  

10) \( \sqrt[48]{250} \)  

11) \( \sqrt{13^2} \)  

12) \( (\sqrt{17})^2 \)  

13) \( (2\sqrt{3})^2 \)  

14) \( (3\sqrt{8})^2 \)  

15) \( (9\sqrt{2})^2 \)  

16) \( 5\sqrt{18} \)  

17) \( 4\sqrt{27} \)  

18) \( 6\sqrt{24} \)  

19) \( 5\sqrt{8} \)  

20) \( 9\sqrt{40} \)
**Algebra Review: Factoring**

Example: (quadratic) \( x^2 + 7x + 12 \)

\[ (x + 3)(x + 4) \]

Factor each expression completely:

1. \( x^2 + 3x \)
2. \( 2x^2 - 10x \)
3. \( x^2 + 3x + 2 \)
4. \( x^2 - 8x + 15 \)
5. \( x^2 + 8x + 16 \)
6. \( x^2 - 6x - 27 \)
7. \( x^2 + 5x - 36 \)
8. \( x^2 - 25 \)
9. \( 9x^2 - 49 \)
10. \( 3x^2 - 5x - 2 \)
11. \( 2x^2 + x - 10 \)
12. \( x^3 - 4x^2 - 21x \)
Algebra Review: Quadratic Equations

Example: \[3x^2 + 14x + 8 = 0\]

\[(3x + 2)(x + 4) = 0\]

\[3x + 2 = 0 \text{ or } x + 4 = 0\]

\[x = -2/3 \quad \text{or} \quad x = -4\]

Solve by factoring:

1) \[x^2 + 5x - 6 = 0\]

2) \[x^2 - 7x - 18 = 0\]

3) \[x^2 = 20x - 36\]

4) \[x^2 + 8x = 20\]

5) \[4x^2 + 15 = 17x\]

6) \[3x^2 - 13x - 10 = 0\]

7) \[6x^2 + 11x - 10 = 0\]

8) \[8x^2 + 10x - 25 = 0\]
Algebra Review: Quadratic Equations (Quadratic Formula)

Example: \(x^2 - 5x + 4 = 0\)  Solve using the quadratic formula

Quadratic formula: \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)

\[
x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(4)}}{2(1)} = \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm \sqrt{9}}{2} = \frac{5 \pm 3}{2} = 4, 1
\]

Solve each equation by the quadratic formula.

1. \(x^2 + 6x + 9 = 0\)
2. \(x^2 - 6x - 40 = 0\)
3. \(x^2 + 8x + 15 = 0\)
4. \(x^2 - 4x - 12 = 0\)
5. \(x^2 + 6x - 4 = 0\)
6. \(2x^2 + 6x + 3 = 0\)
7. \(2x^2 - 7x - 3 = 0\)
8. \(5x^2 - x - 4 = 0\)
9. \(5x^2 + x + 3 = 0\)
10. \(x^2 - 2x + 3 = 0\)
Algebra Review: Writing equations of lines

Slope is the ratio of the change in the y-coordinates over the change in the x-coordinates

\[ m = \frac{\text{rise}}{\text{run}} \]

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

where \((x_1, y_1)\) and \((x_2, y_2)\) are points on the line.

Is the slope of a line the same no matter what two points on the line are given?

**Example:** Find the slope of the line that passes through (3, 6) and (-4, 4).

1. Substitute into the formula.

\[ m = \frac{4 - 6}{-4 - 3} \]

2. Simplify: \( m = \frac{-2}{-7} \) so \( m = \frac{2}{7} \)

**Practice:** Find the slope of the line that passes through each pair of points.

1. (2, 3) and (-4, 8)  
2. (1/4, 1/2) and (3/4, 3/8)

The slope-intercept form of a line is \( y = mx + b \) where \( m \) is the slope and \( b \) is the y-intercept.

**Example:** State the slope and y-intercept of \( y = 2x - 5 \)

**Answer:** Slope = \( m = 2 \)  
y-intercept = \( y = -5 \)
**Practice:** State the slope and y-intercept of the following lines

3. \( y = 4x - 6 \)  
4. \( y = \frac{1}{2} x + 8 \)

**Write an equation of a line using slope-intercept form or point-slope equation.**

Using slope-intercept form

1. Find the y-intercept or “b”. Substitute the slope (m) and the point \((x, y)\) into \(y = mx + b\) and solve for “b”.
2. Substitute the slope “m” and “b” into \(y = mx + b\)

**Example:** Write the equation of the line with slope 5 and passing through (-3,-8)

1. Find the y-intercept “b”.

   Substitute \(m = 5\), \(x = -3\) and \(y = -8\) into \(y = mx + b\)
   
   \[-8 = 5(-3) + b\]
   \[-8 = -15 + b\]
   \[7 = b\]

2. Now substitute into the slope-intercept form of a line

   \[y = 5x + 7\]

Using Point-Slope equation for a line

\((y - y_1) = m(x - x_1)\) where \(m\) is the slope and \((x_1, y_1)\) is a point on the line

**Example:** Write an equation in slope-intercept form for the line that has a slope of -5 through (4, -2)

\((y - (-2)) = -5(x - 4)\)  
Substitute

\((y + 2) = -5x + 20\)  
Simplify

\(y = -5x + 20 - 2\)  
Put in slope-intercept form

\(y = -5x + 18\)  
Simplify

**Practice:** Write the equation of a line in slope-intercept form for the lines with the following slopes and y-intercepts.

5. \(m = \frac{2}{3}\) through (3, -4)
6. \( m = -4 \) through \((1,-3)\)

How do you write an equation of a line given two points?

1. Find the slope using the formula \( m = \frac{y_2 - y_1}{x_2 - x_1} \)

2. Now write the equation of the line using point-slope equation or slope-intercept form.

**Example:** Write the equation of a line through \((2,-1)\) and \((3,-3)\)

1. Find the slope: \( m = \frac{-3 - (-1)}{3 - 2} = \frac{-2}{1} = -2 \)

2. Substitute into the point-slope equation
   \((y - y_1) = m(x - x_1)\)
   \[ y - (-1) = -2(x - 2) \]
   \[ y + 1 = -2x + 4 \]
   \[ y = -2x + 3 \]

**Practice:** Write the equation for the following lines through:

7. \((3,-6)\) and \((6,2)\)

8. \((-7,2)\) and \((-3,5)\)