

AP CALCULUS BC SEMINAR SUMMER WORK

Be prepared to turn in the work for this assignment on the first day of school. Write all of your work and answers on separate paper. We will go over the assignment, make corrections, and have a quiz on it by the end of the first week.

I. Calculator Basics

1. Determine which of the following gives a complete graph for the indicated equation:

a) $y = -x^3 + 8x^2 - x + 5$

b) $f(x) = \frac{3x^2 + x - 5}{x^2 + 1}$

(i) $[-10,10] \times [-10,10]$

(i) $[-10,10] \times [-10,10]$

(ii) $[0,10] \times [-10,80]$

(ii) $[-2,20] \times [-20,20]$

(iii) $[-5,10] \times [-10,80]$

(iii) $[0,20] \times [-5,5]$

(iv) $[-50,50] \times [-100,100]$

(iv) $[-5,20] \times [-5,5]$

2. An open box is to be made from cutting squares of side s from each corner of a piece of cardboard 25" by 30".

- Write an expression for the volume, V , of the box in terms of s .
- Graph of $V(s)$ using your graphing calculator. Identify the domain and range of this graph.
- What domain and range make sense in this problem situation?
- Find the value of s that will give the maximum volume of the box.
- What value(s) of s will give a volume of 1225 cubic inches?

II. Cartesian Plane Basics

3. Determine the slope, the length, and the midpoint of the segment with endpoints $(1,-2)$ and $(3,2)$.

4. For what values of k is $5x + ky = 3$ parallel to $2x - 3y = 5$? For what values of k are the two lines perpendicular?

5. Suppose you a Lear jet for one day. Knowing that Swissair rents a Lear jet with a pilot for \$2000 per day plus \$1.75/mile and that Air France rents a Lear jet with a pilot for \$1500 per day plus \$2.00/mile, find the following:

- For each company, write a function for cost as a function of distance traveled.
- If cost were the only consideration, when would you rent from Air France?

III. Basic Functions and Transformations

6. You should be very familiar with the following functions and be able to envision them in your mind without using a graphing calculator. You do not need to write anything for this problem **EXCEPT for parts g and h, where you must graph the function and write its domain and range..**

- constant function: $f(x) = C$, C is a constant.
- linear function: $f(x) = mx + b$

- c. quadratic function: $f(x) = ax^2 + bx + c$
- d. polynomial function: $f(x) = ax^n + bx^{n-1} + \dots + mx + n$
- e. rational function: $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials
- f. radical function: $f(x) = \sqrt{x}$ and $g(x) = \sqrt[3]{x}$
- g. exponential function: Graph $f(x) = e^x$ Write the domain and range.
- h. natural log function: Graph $f(x) = \ln x$ Write the domain and range.
- i. absolute value function: $f(x) = |x|$

7. Using transformations, describe how the graph of each function can be obtained from the graph of

$$f(x) = \sqrt{x}, g(x) = \frac{1}{x}, h(x) = |x|, k(x) = x^3, l(x) = \ln x, \text{ OR } m(x) = e^x.$$

- a) $y = .5(x-4)^3 + 2$
- b) $y = |x+2| - 1$
- c) $y = -\sqrt{3-x} + 4$
- d) $y = 2\ln(x+3) - 5$
- e) $y = e^x - 3$

IV. Trig Review

Your life (and this class) will be a lot easier next year if you will remember the trig values for special angles, the basic shape of the sine, cosine, and tangent functions, their domains and ranges, and these trig identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin 2x = 2 \sin x \cos x$$

8. **You must be able to do the following WITHOUT a calculator.** We will always use radians, not degrees.

Evaluate:

- a) $\sin \pi$
- b) $\cos\left(\frac{3\pi}{2}\right)$
- c) $\tan\left(\frac{5\pi}{4}\right)$
- d) $\sin\left(\frac{4\pi}{3}\right)$
- e) $\cos\left(\frac{7\pi}{4}\right)$
- f) $\sin\left(\frac{2\pi}{3}\right)$
- g) $\tan\left(\frac{5\pi}{6}\right)$
- h) $\sin\left(\frac{11\pi}{6}\right)$

9. Solve each of the following on the interval $[0, 2\pi)$.

- a) $2 \sin x - \sqrt{3} = 0$
- b) $\cos^2 x = 2 \sin x - 2$

10. Solve each of the following on the interval $[0, 2\pi)$.

- a) $\sin^2 x < 2 \cos x + 1$
- b) $2 \sin x \cos x + \sqrt{2} \cos x < 0$

V. Exponents, Radicals, and Factoring

11. Simplify each of the following expressions.

a) $\left(\frac{2}{3}x^{-3}\right)(15x^7)$ b) $x^3(2yz^2)^3$ c) $\frac{(3x^3)(4x^5)}{(x^2)^3}$

d) $\frac{(2y^4)(3y^2)^2}{(y^3)^4}$ e) $(3a^{-2}b^3)^{-3}$

12. Simplify by removing all possible factors from the radical.

a) $\sqrt{9a^8b}$ b) $\sqrt[3]{24a^4b^8}$ c) $\sqrt{\frac{75}{a^6}}$

13. Factor each of the following completely.

a) $6x^3y^2 + 15x^2y^5 - 30x^7y^4z$ b) $16y^2 - 9$
c) $4x^{16} - 9y^6$ d) $6x^2 + 7x - 20$
e) $3x^2 - 5x + 2$ f) $x^3 - x^2 + 3x - 3$
g) $3a^3 + 3a^2 - 27a - 27$ h) $x^2 + 4x + 4 - 9y^2$