AB CALCULUS SUMMER WORK

I. Calculator Basics

1. Determine which of the following gives a complete graph for the indicated equation:

a) $y = -x^3 + 8x^2 - x + 5$

(i) [-10,10] X [-10,10]
(ii) [0,10] X [-10,80]
(iii) [-5,10] X [-10,80]
(iv) [-50,50] X [-100,100]

b) $f(x) = \dfrac{3x^2 + x - 5}{x^2 + 1}$

(i) [-10,10] X [-10,10]
(ii) [-2,20] X [-20,20]
(iii) [0,20] X [-5,5]
(iv) [-5,20] X [-5,5]

2. Find the largest and smallest values of each of the following functions on their given intervals.

a) $f(x) = 2^x + x^2$ [-4,1]

b) $y = (\cos x)^x$ [-1.5,4.75]

c) $f(x) = \dfrac{3x^2 + x - 5}{x^2 + 1}$ [2,4]

d) $g(x) = \dfrac{1}{\sqrt{4 - x^2}}$ (-2,2)

3. Graph $y = \dfrac{x^2 - 9}{x - 3}$ on your graphing calculator.

a) Explain why this graph appears to be a linear function rather than a curve with a vertical asymptote at $x = 3$.
b) Zoom in on the graph around $x = 3$. Give the viewing window when you first see the empty pixel at $x = 3$ **or** you notice the graph getting "jagged' around there.
c) If you had to fill in a y-value when $x = 3$, what would it be?

4. An open box is to be made from cutting squares of side *s* from each corner of a piece of cardboard 25" by 30".

a) Write an expression for the volume, *V*, of the box in terms of *s*.
b) Graph of *V(s)* using your graphing calculator. Identify the domain and range of this graph.
c) What domain and range make sense in this problem situation?
d) Find the value of *s* that will give the maximum volume of the box.
e) What value(s) of *s* will give a volume of 1225 cubic inches?

II. Cartesian Plane Basics

5. Determine the slope, the length, and the midpoint of the segment with endpoints (1,-2) and (3,2).

6. For what values of *k* is $5x + ky = 3$ parallel to $2x - 3y = 5$? For what values of *k* are the two lines perpendicular?

7. Plot the line $2x - 5y = 10$ indicating x- and y-intercepts. Be sure to label your axes to indicate your scale.

Problems 1-18 adapted from Adrienne Hestenes, Xavier College Prep, Phoenix, AZ

8. Find the distance from point P(1,2) to line *m*: $x+2y=3$.

9. Suppose you a Lear jet for one day. Knowing that Swissair rents a Lear jet with a pilot for $2000 per day plus $1.75/mile and that Air France rents a Lear jet with a pilot for $1500 per day plus $2.00/mile, find the following:

a) For each company, write a function for cost as a function of distance traveled.
b) If cost were the only consideration, when would you rent from Air France?

10. Find the line that passes through the point (-1,3) and the point of intersection of the lines $x+3y=1$ and $2x-y=-5$. Leave your answer in point slope form ($y-y_1=m\left(x-x_1\right)$).

III. Basic Functions and Transformations

11. You should be very familiar with the following functions and be able to envision them in your mind without using a graphing calculator.

1. constant function: $f(x)=C,\quad C$ is a constant.
2. linear function: $f(x)=mx+b$
3. quadratic function: $f(x)=ax^2+bx+c$
4. polynomial function: $f(x)=ax^n+bx^{n-1}+\ldots+mx+n$
5. rational function: $f(x)=\dfrac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials
6. radical function: $f(x)=\sqrt{x}$ and $g(x)=\sqrt[3]{x}$
7. exponential function: $f(x)=e^x$ Know the domain and range.
8. natural log function: $f(x)=\ln x$ Know the domain and range.
9. absolute value function: $f(x)=|x|$

12. Using transformations, describe how the graph of each function can be obtained from the graph of

$f(x)=\sqrt{x}$, $g(x)=\dfrac{1}{x}$, $h(x)=|x|$, $k(x)=x^3$, $l(x)=\ln x$, **OR** $m(x)=e^x$.

a) $y=.5\left(x-4\right)^3+2$
b) $y=|x+2|-1$
c) $y=-\sqrt{3-x}+4$
d) $y=2\ln\left(x+3\right)-5$
e) $y=e^x-3$

13. Remembering that for even functions, $f(-x)=f(x)$ and that for odd functions, $f(-x)=-f(x)$, show whether the following functions are even, odd, or neither:

a) $y=x^4$
b) $y=x-x^4$
c) $y=\dfrac{1}{x^2-4}$
d) $y=-2x^3+4x$

IV. Trig Review

Your life (and this class) will be a lot easier next year if you will remember the trig values for special angles, the basic shape of the sine, cosine, and tangent functions, their domains and ranges, and these trig identities:

$\sin^2\theta+\cos^2\theta=1$ $\qquad$ $\sin 2x=2\sin x\cos x$

14. **You must be able to do the following WITHOUT a calculator.** We will always use radians, not degrees.

Evaluate:

a) $\sin\pi$ $\qquad$ b) $\cos\left(\frac{3\pi}{2}\right)$ $\qquad$ c) $\tan\left(\frac{5\pi}{4}\right)$ $\qquad$ d) $\sin\left(\frac{4\pi}{3}\right)$

e) $\cos\left(\frac{7\pi}{4}\right)$ $\qquad$ f) $\sin\left(\frac{2\pi}{3}\right)$ $\qquad$ g) $\tan\left(\frac{5\pi}{6}\right)$ $\qquad$ h) $\sin\left(\frac{11\pi}{6}\right)$

15. Convert the following to radians:

a) $510°$ $\qquad$ b) $120°$ $\qquad$ c) $135°$ $\qquad$ d) $-210°$

16. Identify which of the six trig functions are

a) even
b) odd
c) neither

17. Which two equations have the same graph?

a) $y=\sin x$ $\qquad$ b) $y=\sin(-x)$ $\qquad$ c) $y=\cos x$ $\qquad$ d) $y=-\sin x$ $\qquad$ e) $y=-\cos x$

18. Consider the function $y=\sqrt{\frac{1+\cos 2x}{2}}$.

a) Can *x* take on any real value?
b) How large can $\cos 2x$ become? How small?
c) How large can $\frac{1+\cos 2x}{2}$ become? How small/
d) What are the domain and range of the original function?

19. Solve each of the following on the interval $[0,2\pi)$.

a) $2\sin x-\sqrt{3}=0$ $\qquad$ b) $\cos^2 x=2\sin x-2$

20. Solve each of the following on the interval $[0,2\pi)$.

a) $\sin^2 x<2\cos x+1$ $\qquad$ b) $2\sin x\cos x+\sqrt{2}\cos x<0$

V. Exponents, Radicals, and Factoring

21. Simplify each of the following expressions.

a) $\left(\frac{2}{3}x^{-3}\right)\left(15x^{7}\right)$ b) $x^{3}\left(2yz^{2}\right)^{3}$ c) $\frac{\left(3x^{3}\right)\left(4x^{5}\right)}{\left(x^{2}\right)^{3}}$

d) $\frac{\left(2y^{4}\right)\left(3y^{2}\right)^{2}}{\left(y^{3}\right)^{4}}$ e) $\left(3a^{-2}b^{3}\right)^{-3}$

22. Simplify by removing all possible factors from the radical.

a) $\sqrt{9a^{8}b}$ b) $\sqrt[3]{24a^{4}b^{8}}$ c) $\sqrt{\frac{75}{a^{6}}}$

23. Factor each of the following completely.

a) $6x^{3}y^{2}+15x^{2}y^{5}-30x^{7}y^{4}z$ b) $16y^{2}-9$

c) $4x^{16}-9y^{6}$ d) $6x^{2}+7x-20$

e) $3x^{2}-5x+2$ f) $x^{3}-x^{2}+3x-3$

g) $3a^{3}+3a^{2}-27a-27$ h) $x^{2}+4x+4-9y^{2}$