Name____

Pre-Calculus Summer Packet 2020-2021 LBHS

This packet is to help you review various topics that are considered to be prerequisite knowledge upon entering Honors PreCalculus. It is **due on the 1st day of the semester**!!! Your first grade of the semester will include information from this packet. You will have a Quiz on Day 5 from review material on this packet.

- ***Show all of your work on separate sheets of paper NEATLY and organized!***
- ***All questions will be graded for correctness.***
- ***Show all work for credit.***
- ***Questions with NO work will receive NO credit!!***
- ***Box your answers!***
- ***NO CALCULATOR UNLESS OTHERWISE STATED!***

I. Geometry Topics

- **Midpoint formula**: $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ - **Median of a Triangle**: A segment from a vertex to the midpoint of the opposite side. - **Angle Bisector of a Triangle**: A segment from a vertex which bisects the angle. - **Perpendicular Bisector**: A line passing through the midpoint of and perpendicular to a segment. - **Altitude of a Triangle**: A segment from a vertex perpendicular to the opposite side.	- Equations of Lines: 1. Slope-intercept: $y = mx + b$ where $m=\frac{\Delta y}{\Delta x}$ (Note: here, Δ means "change in." For example, $\Delta y = y_2 - y_1$) 2. Point-slope: $y - y_1 = m(x - x_1)$ 3. Standard form: $Ax + By + C = 0$ - Distance Formula: $d=\sqrt{(\Delta x)^2+(\Delta y)^2}$

Directions - State all linear equations in Slope-Intercept Form unless otherwise stated.

1. Write the equation of the line parallel to the line $4x-6y=-1$ containing the *x*-intercept of $3x-2y=12$.

2. Write an equation of the line with x-intercept of 3 and y-intercept of 5.

3. Write the equation of the line through $(2,-4)$ and perpendicular to $x-2y=7$.

4. Find the value of "*a*" if a line containing the point $(a,-3a)$ has a *y*-intercept of 7 and a slope of $-\frac{2}{3}$.

II. Quadratics/Polynomials

A. Factoring—Strategies to try when factoring:

- Look for a common factor - Difference of two squares: $a^2 - b^2 = (a-b)(a+b)$ - Perfect square trinomial: $a^2 \pm 2ab + b^2 = (a \pm b)^2$ - Factorable trinomial (Target Sum/Target Product)	- Guess and Check - Grouping - Sum/Difference of Cubes ○ $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ ○ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

1. *Directions - Factor completely each of the following:*

a. $4x^2 + 27x + 35$ b. $-28y^2 + 7t^2$ c. $x^3 - 2x^2 - 9x + 18$

e. $4x^3 + 8x^2 - 5x - 10$ f. $8x^3 - 27$ g. $x^4 - 16$ h. $x^2 + 9$

B. Equations - Since the following are equations, we can now go a step further and solve for *x* by factoring or using the quadratic formula.

2. *Directions - Solve each of the following:*

a. $-3x^2 - 5x + 12 = 0$ b. $4y - 2 = y^2$ c. 225-b^2 = 0 d. $x^2 = x$

e. $4y^2 = 25$ f. $(y-3)^2 = 10$ g. $3x^2 + 2x - 1 = 0$ h. $3x^3 + 3x^2 - 27x - 27 = 0$

A. Graphing - To graph a quadratic equation in standard form, $y = ax^2 + bx + c$, find the important points of the graph by following the steps:

- ***Y-intercept:*** If a point is the *y*-intercept of the curve, then that is the point at which the graph crosses the *y*-axis. Since this point is on the *y*-axis, then the *x*-coordinate must be 0. Substitute zero in for *x* and solve for *y*. - ***Vertex:*** *x*-coordinate of the vertex: $x = -\frac{b}{2a}$. *y*-coordinate of the vertex: substitute the value found for the *x*-coordinate into the original equation and solve for y. - ***X-intercepts:*** If a point is an *x*-intercept of the curve, then it is a point at which the graph crosses the *x*-axis. Since these points are on the *x*-axis, then the *y*-coordinates must be 0. Substitute zero in for *y* and solve for *x* by factoring or using the quadratic formula.

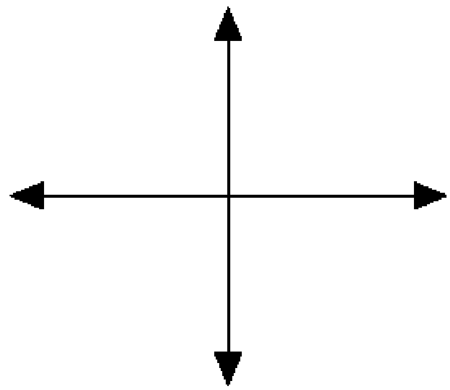
*No calculator, but you should also be able to graph with the use of your calculator.

3. *Directions – Given* $y = -3x^2 + x + 2$, *find and graph.*

a. y-intercept

b. vertex

c. x-intercepts



III. Systems

Substitution or Linear Combination (Elimination) can be used to solve systems of equations.

- If there is a solution to the system, then the equations are representing intersecting lines.
- If both variables cancel out and an equation is formed that is never true, then there is no solution and the lines never intersect. Lines that never intersect are parallel lines.
- If both variables cancel out and an equation is formed that is always true, then there areinfinitely many solutions and the equations must represent the same line.

Directions - Solve each of the following.

- *Explain what the solution tells us about the lines represented by the equations.*
- *No calculator, but you need to be able to solve with the use of a calculator as well.*

1. $\begin{cases} 3x - 4y = 2 \\ -x + 3y = 1 \end{cases}$ Solution:_________________________ Explanation:	2. $\begin{cases} -x + y = 3 \\ 2x - 2y = -6 \end{cases}$ Solution:_________________________ Explanation:

IV. Exponents

Directions - Simplify using only positive exponents and no calculator!!!

Properties:	$a^m \cdot a^n = a^{m+n}$	$\left(a^m\right)^n = a^{m \cdot n}$	$a^{p/r} = \sqrt[r]{a^p}$
$a^0 = 1, a \neq 0$	$a^{-n} = \dfrac{1}{a^n}$	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$	$\dfrac{a^m}{a^n} = a^{m-n}$
	$a^{-p/r} = \dfrac{1}{\sqrt[r]{a^p}}$	$\left(\dfrac{a}{b}\right)^{-m} = \left(\dfrac{b}{a}\right)^m = \dfrac{b^m}{a^m}$	$\dfrac{a^n b^{-n}}{c^{-m}} = \dfrac{a^n c^m}{b^n}$

1. $\left(\dfrac{81}{64}\right)^{-\frac{1}{2}}$

2. $\left(27^{-2}\right)^{-\frac{1}{3}}$

3. $\dfrac{\left(3x^2\right)^{-1}}{6x^{-3}}$

4. a. -2^4 , b. $(-2)^4$

5. $\dfrac{3^{-5} \cdot 3^{-10}}{3^{2}}$

6. $\left(4^{-1} + 2^{-1}\right)^{2}$

- hint 1: $\left(a^{-m} + a^{-n}\right)^{p} \neq a^{-mp} + a^{-np}$
- hint 2: Apply the negative exponent property to each term, then get a common denominator, then add.

7. a. $(13y)^{-1}$, b. $13y^{-1}$

8. $8^{-1} \times 8^{0}$

V. Logarithms

Given $\log_b a = x$ if and only if $b^x = a$, where $b > 0$, but $b \neq 1$ and $a > 0$

Directions: - Solve for x.

1. $3\log_2 x = 12$
2. $\log_5 125 = x$
3. $3 + 4\log_x 4 = 5$

VI. Rational Expressions

Directions - Simplify:

1. Hint: factor and cancel!

a. $\dfrac{a^3 - a^2 - 6a}{a^2 - 9}$ b. $\dfrac{a^3 - 2a^2 + a - 2}{2 - a}$ c. $\dfrac{z^3 + 8}{z^2 - 2z + 4}$ d. $\dfrac{4ab^2}{ab^2 + a}$ e. $\dfrac{x^2 + 6x + 8}{x^2 - 4}$

2. Perform the operation and write the result in reduced form.

a. $\dfrac{a-1}{a} \text{g} \dfrac{a^2}{a^2 - 1}$ b. $\dfrac{a^2 - 9}{a+2} \text{g} \dfrac{a^2 + 4a + 4}{a^2 - a - 6}$ c. $\dfrac{x+4}{x^2} \div \dfrac{x^2 - 16}{x}$

3. Hint: get a common denominator!

a. $\dfrac{2}{2x+1} - \dfrac{5}{(2x+1)^2}$ b. $3 - \dfrac{4}{x+2}$ c. $\dfrac{1}{x^2 - 3x + 2} + \dfrac{2}{x^2 - 4}$ d. $\dfrac{3}{x-2} + \dfrac{5}{2-x}$ e. $\dfrac{y}{y^2 + 4y + 4} + \dfrac{3}{y^2 + y - 2}$

4.Hint: get a common denominator in the numerator and multiply by the reciprocal, or multiply by LCD/LCD.

a. $\dfrac{1-\dfrac{y}{3}}{3-y}$

b. $\dfrac{\dfrac{1}{y+1}-\dfrac{1}{y}}{\dfrac{1}{y+1}}$

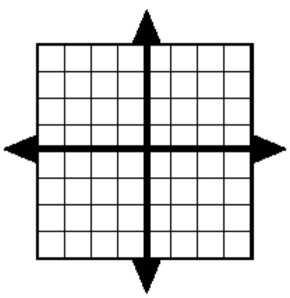
c. $\dfrac{\dfrac{x}{x-1}+1}{\dfrac{x+2}{x}}$

VII. Quick Graphs:

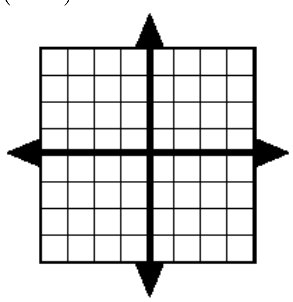
Directions - Graph each of the following.

- *If you don't remember, use your graphing calculator to help you determine the patterns. But you need to be able to do these graphs without your calculator!*

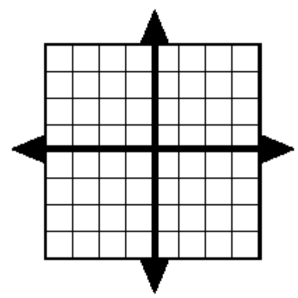
1. $y=\sqrt{x-2}-3$



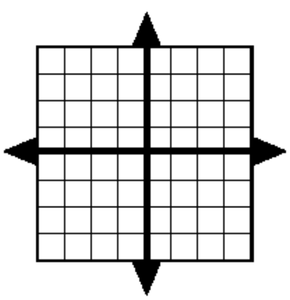
2. $y=(x+2)^2+1$



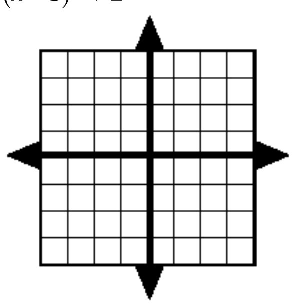
3. $y=|x|-1$



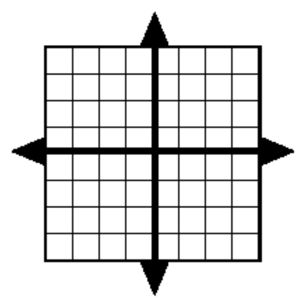
4. $y=\sqrt[3]{x+1}-2$



5. $y=(x-3)^3+2$



6. $y=x+3$



VIII. Simplifying Radicals

To Simplify a radical:

- find the largest perfect square which will divide evenly into the number under your radical sign.
- If the number under your radical cannot be divided evenly by any of the perfect squares, your radical is already in simplest form and cannot be reduced further.

You should be able to do the following operation in your head!!!!!

Example: $\sqrt{48}$

- write the number appearing under your radical as the product (multiplication) of the perfect square and your answer from dividing: $\sqrt{48}=\sqrt{16\cdot 3}$

- give each number in the product its own radical sign: $\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3}$
- reduce the "perfect" radical which you have now created: $\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$
- you now have your answer: $4\sqrt{3}$

Directions – Simplify each of the radicals without a calculator.

a. $\sqrt[3]{128}$ b. $\sqrt{9a^8b^3}$ c. $\sqrt[3]{24a^4b^8}$ d. $\sqrt{\dfrac{75}{a^6}}$ e. $\dfrac{5}{\sqrt{7}}$ f. $\dfrac{3y}{4-\sqrt{7}}$

2. Solve the following radical equations.

a. $\sqrt{7-5x} = 8$ b. $\sqrt{2y+15} = \sqrt{4y+1}$ c. $\sqrt[3]{3x+4} + 2 = 0$ d. $(x-3)^{\frac{2}{3}} = 4$

X. Domain and Range

- Domain of a function $f(x)$: the set of all real numbers variable x can take such that the expression defining the function is real. The domain can also be given explicitly. Values not in the domain are those that yield division by zero or that yield a negative under an radical with even root.

  Ex1) $f(x) = \dfrac{2}{x-3}$. Domain: $\{x | x \neq 3\}$ or $(-\infty, 3), (3, \infty)$

  Ex2) $f(x) = \sqrt{x}$. Domain: $\{x | x \geq 0\}$ or $[0, \infty)$

  Ex3) $f(x) = \{(1,2), (2,-3)(5,2), (6,7)\}$. Domain: $\{x | x = 1,2,5,6\}$
- Range of a function $f(x)$: the set of all y values that the function takes when x takes values in the domain. (This is more easily determined from the graph)

  Ex1) $f(x) = \dfrac{2}{x-3}$. Range: $\{y | y \neq 0\}$ or $(-\infty, 0), (0, \infty)$

  Ex2) $f(x) = \sqrt{x}$. Range: $\{y | y \geq 0\}$ or $[0, \infty)$

  Ex3) $f(x) = \{(1,2), (2,-3)(5,2), (6,7)\}$. Range: $\{y | y = -3,2,7\}$
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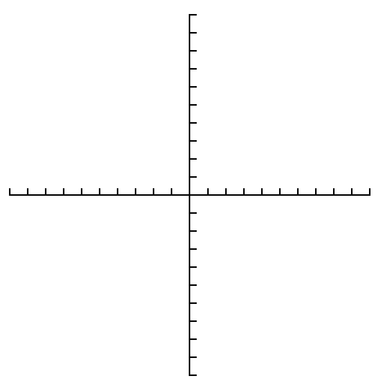
Directions: Find the Domain of the following functions.

a. $f(x) = x^2 + 4$ b. $f(x) = \dfrac{1}{x} + \dfrac{2}{x+2}$ c. $g(x) = \dfrac{x}{x^2 - 5x}$ d $h(x) = \sqrt{4-x}$ e. $g(x) = \sqrt{x^2+1}$

XI. Miscellaneous Problems:

1. Graph the function $y = \begin{cases} -x-2 & \text{for } -2 < x \leq -1 \\ -x^2 & -1 < x \leq 1 \\ x+2 & 1 < x \leq 2 \end{cases}$



2. Solve the exponential equations.

a. $\left(\frac{1}{2}\right)^{\frac{x}{3}} = \frac{1}{4}$

b. $3 \cdot 4^{\frac{x}{2}} = 96$

c. $e^{x^2+5x} = e^{-6}$

d. $3\left(5^{-x/4}\right) = 15$