

AP Calculus Summer Work

I am so excited to have you in AP Calculus this year! Calculus brings all the math you've learned throughout the years together and allows you to evaluate and justify real world situations mathematically. In the following pages you will find a few review topics we will cover in the first few days of the year. We will briefly review the topics and have some time to revise your solutions before corrections. Your goal this summer is to complete these worksheets to the best of your ability, reviewing topics that will be imperative to your success next year.

At the beginning of each section are links for helpful videos to help you with corresponding sections in the summer work packet. Use these videos as references to recall material covered in PreCalculus. Some topics may be from courses prior to PreCalculus, so they may be more challenging to recall. Do your best to complete, or at least make a solid attempt in all of the sections. Come ready with questions and we will fill in the gaps together!

If you have any questions, please feel free to email me over the summer break and I will get back to you as quickly as possible!

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Linear Review

Video resources

Horizontal and vertical lines - <https://www.youtube.com/watch?v=2GYbX7PbmzA>

Parallel and perpendicular line - <https://www.youtube.com/watch?v=LTb2-LE7StE>

Line tangent to a circle - https://www.youtube.com/watch?v=ucEghF7_HvU

Show all work. No Calculator

1. Write an equation of the (a) vertical line and (b) horizontal line passing through the point $(-7, \pi)$.

2. Write an equation of the line in **slope-intercept form** passing through the points $(-3, 4)$ and $(5, -1)$.

3. Write an equation of the line through the point $(4, \frac{1}{2})$ that is

(a) parallel to $4x - 3y = 6$

(b) perpendicular to the line $4x - 3y = 6$

(c) find both the x - and y - intercepts of the line found in part (b).

4. Find the value of x for which line through $(-8, -3)$ and $(x, 4)$ has a slope of 3.

5. Write the equation of the line that is tangent to the circle at the given point.

$$(x - 2)^2 + (y - 1)^2 = 25; (6, -2)$$

6. Mr. Wenzel leaves school in his truck along Loop 337 at time $t = 0$ traveling at 45 mph.

(a) Write an expression $d(t)$ for the distance Mr. Wenzel travels from school beginning at $t = 0$ hours.

(b) Graph $y = d(t)$

(c) What is the slope of the graph in part (b)? What does it have to do with the truck?

(d) Create a scenario in which t could have negative values.

(e) Create a scenario in which the y -intercept of $y = d(t)$ could be 15.

Rational and Radical Functions

Video resources

Simplifying rational expressions - <https://www.youtube.com/watch?v=7Uos1ED3KHI>

Adding and subtracting rational expressions - https://www.youtube.com/watch?v=y_DweTAEYWk

Multiplying and dividing rational expressions - https://www.youtube.com/watch?v=x_5hDLe8ULO

Solving rational expressions - https://www.youtube.com/watch?v=1fR_9ke5-n8

<https://www.youtube.com/watch?v=tynJHA7wFzA>

Solving radical equations - <https://www.youtube.com/watch?v=g3rzugglgw>

Simplify and determine where the function is undefined

$$1. \frac{4x+16}{2x+6} \square \frac{x^2+2x-3}{x+4}$$

$$2. \frac{x+3}{x-1} \square \frac{x^2-2x+1}{x^2+5x+6}$$

$$3. \frac{x^2-2x-8}{x^2-2x-15} \div \frac{2x^2-8x}{2x^2-10x}$$

$$4. \frac{\frac{12}{x+3}}{\frac{x^2+1}{x-2}}$$

$$5. \frac{x+4}{x^2-x-12} + \frac{2x}{x-4}$$

$$6. \frac{3x^2-1}{x^2-3x-18} - \frac{x+2}{x-6}$$

Solve. Check your solution.

$$7. \frac{3-x}{x^2-9} = -3$$

$$8. x = \frac{3}{x} + 2$$

$$9. \frac{4}{x^2-4} = \frac{1}{x-2}$$

Solve each equation.

$$10. \sqrt{x+6} = 7$$

$$11. \sqrt{x+4} = 3\sqrt{x}$$

$$12. \sqrt[3]{x-6} = \sqrt[3]{3x+24}$$

$$13. \sqrt{-14x+2} = x-3$$

$$14. (x+4)^{\frac{1}{2}} = 6$$

Exponential and Logarithmic Functions

Video resources

Log properties – <https://www.youtube.com/watch?v=mQTWzLpCcW0>

<https://www.youtube.com/watch?v=Jtv9Lnf7Zw8>

Expanding logarithms - <https://www.youtube.com/watch?v=Olz-5MyJA3g>

Condensing logarithms - <https://www.youtube.com/watch?v=luRrOlsB4cY>

Solving log equations – <https://www.youtube.com/watch?v=Kv2iHde7Xgw>

<https://www.youtube.com/watch?v=eTWCARmrzJ0>

Solving exponential equations - <https://www.youtube.com/watch?v=9tutJ5xRWg>

Expand each logarithm

1. $\log\left(\frac{x^4}{y}\right)^3$

2. $\log\frac{\sqrt{xy^3}}{z^2}$

3. $\log\left(\frac{\sqrt[3]{xy^6}}{(zk)^2}\right)$

Condense each logarithmic expression and simplify when possible

4. $\log 2 + 3 \log x - 2 \log 4x$

5. $4 \log z + 4 \log x + 16 \log y$

6. $\log u - \log x - 3 \log w + 2 \log y$

7. $\left(\frac{1}{2} \log x + \frac{1}{2} \log y\right) - (\log 2 - 4 \log z)$

Solve the following exponential and logarithmic equations

8. $12^{2x-8} = 15$

9. $25^x = 125^{x-2}$

10. $\left(\frac{1}{32}\right)^{2x} = 64$

11. $\log(t+3) + \log(t) = 1$

12. $\log_2(t+1) + \log_2(t-1) = 5$

13. $-2 = \log(2) - \log(3+x)$

Simplify the following logarithms

14. $\ln e^{7x} + \ln e^{x+4} - e^{\ln x}$

15. $e^{3 \ln x} - 2 \ln e^x$

Trig Review

Video resources

Unit circle – https://www.youtube.com/watch?v=V5ArB_GFGYQ
<https://www.youtube.com/watch?v=wT4xMAssvDk>

Graphing sine and cosine – <https://www.youtube.com/watch?v=bvTR9Xbl1co>
<https://www.youtube.com/watch?v=KPvon2ufyNs>

Graphing other trig equations – https://www.youtube.com/watch?v=fo_q9mEAFp4
<https://www.youtube.com/watch?v=sKoR8KHx4uU>

Fundamental trig identities – <https://www.youtube.com/watch?v=ep5vjIY5kqE>
https://www.youtube.com/watch?v=UFb_LQxeGfg

Sum and difference identities – https://www.youtube.com/watch?v=d0iUBZz_WIE
<https://www.youtube.com/watch?v=hdSvvW0hpeE>

Double and half angle identities – <https://youtu.be/-Wu6ekO2Sdw>
<https://youtu.be/bNoJC7R2c0I>
<https://youtu.be/Xlel4XLomn4>

Trig equations - <https://youtu.be/8wa-PysIIDw>
https://youtu.be/gC_pqtQ9z0U
<https://youtu.be/mzJZRInvevc>
https://youtu.be/JioF8Vm_M60
<https://youtu.be/hHUSsGP02Ik>

Find the exact value of the following.

1. $\csc(-270^\circ)$ 2. $\sec\left(\frac{11\pi}{3}\right)$ 3. $\tan(480^\circ)$ 4. $\sin\left(-\frac{10\pi}{3}\right)$

Determine the amplitude, period, intervals, phase shift, and midline for each. Sketch the graph of each function for ONE CYCLE.

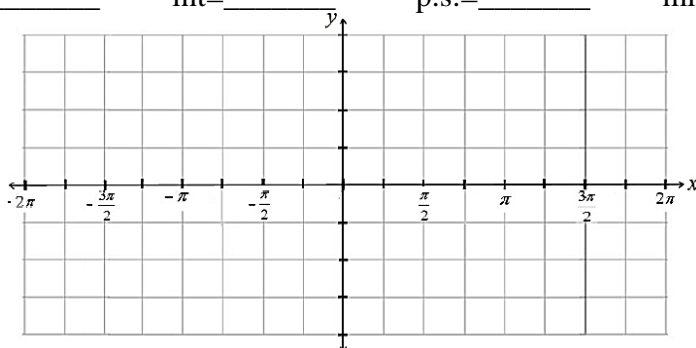
5. $f(x) = -2\sin\left(3x - \frac{\pi}{2}\right) - 1$

amp=_____ pd=_____ int=_____ p.s.=_____ mid=_____



6. $f(x) = 2 + \cos\frac{1}{2}(x + 2\pi)$

amp=_____ pd=_____ int=_____ p.s.=_____ mid=_____



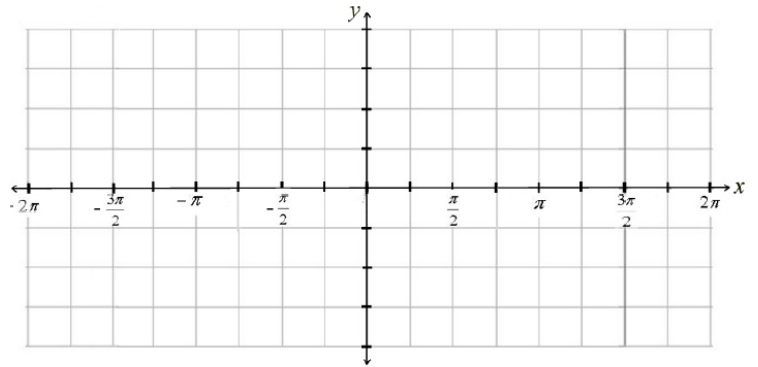
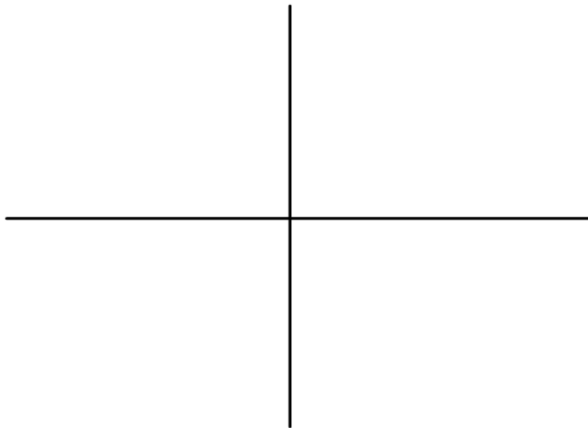
Determine the period, x-intercepts, and/or asymptotes for the following functions. Graph at least one cycle in either direction for tangent and cotangent, and one positive cycle for secant and cosecant.

7. $f(x) = -\tan 3x$

8. $f(x) = 5\cot \frac{2}{3}x$

pd = _____ x-int = _____ asm = _____

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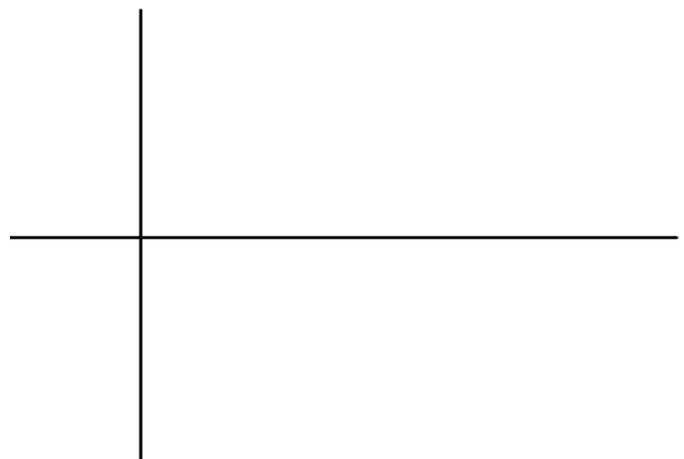
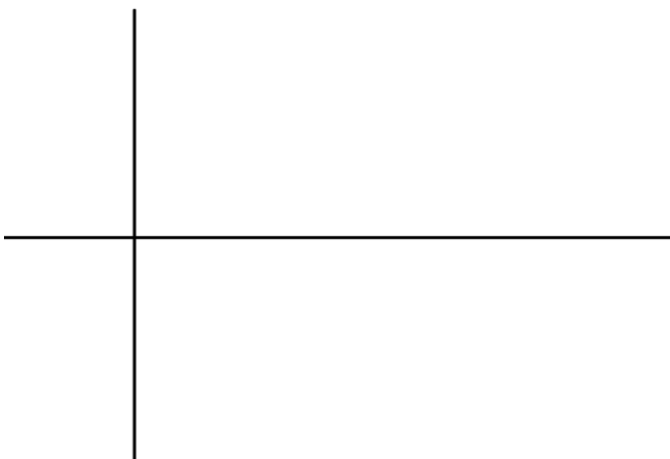


9. $f(x) = \frac{1}{2} \csc 3x$

10. $f(x) = \sec \frac{1}{2}x$

pd = _____ asm = _____

pd = _____ asm = _____



11. Prove $\tan x \sin x + \cos x = \sec x$

12. Prove $\frac{1 - \cot^2 x}{\tan^2 x - 1} = \cot^2 x$

13. Prove $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

14. Prove $\cos 2\theta = \frac{\sin 2\theta - \tan \theta}{\tan \theta}$

15. Given $\cos\theta = -\frac{3}{5}$ with $\pi < \theta < \frac{3\pi}{2}$, find the following.

a. $\sin 2\theta$

b. $\cos 2\theta$

c. $\sin \frac{\theta}{2}$

d. $\tan \frac{\theta}{2}$

Find the exact value of each expression using a sum or difference identity.

16. $\sin 165^\circ$

Find the exact value of each expression using a half angle identity.

17. $\cos 105^\circ$

Find all of the solutions of each equation.

1. $3\sin\theta = \sin\theta - \sqrt{3}$

2. $5\cos\theta = 3\cos\theta - 1$

Solve each equation for the given domain.

3. $\cos^2\theta - 2\cos\theta = -1$ for $0^\circ \leq \theta \leq 360$

4. $4\sin^2\theta = 3\sin\theta$ for $0^\circ \leq \theta \leq 360^\circ$

Solve each equation for the given domain. Use trigonometric identities.

5. $\cos^2\theta = \sin^2\theta + \sin\theta$ for $0^\circ \leq \theta \leq 360^\circ$

6. $\cos 2\theta + \sin\theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$