

Calculus Honors Summer Assignment

Welcome to Calculus Honors! As we prepare for an exciting and challenging year ahead, it's important to ensure that our minds stay sharp and ready for the rigors of Calculus. Strong Algebra, Geometry, and Trigonometry skills are absolutely vital to successfully completing Calculus. Our discussion of Calculus will pull examples from many areas in mathematics. Comfort in the necessary skills from your previous courses will prove immensely helpful as we study these examples and develop our mathematical understanding of real world phenomena.

The intent of this assignment is to do a review of concepts to make sure you are prepared for Calculus Honors. Work through each section until you are comfortable with the content the section covers. Write **all** of your work on a separate piece of paper. **Do NOT wait until the last minute to complete this assignment!** However, you should not spend your entire summer working through these problems. Take time to relax and recharge over summer, but budget enough time so you are not rushing through this assignment. Managing your time on this assignment will play a major role in setting the tone for how you manage your time on assignments moving forward.

Try your best to complete each section. We will discuss these topics over the first few days of the year. You will be assessed on this material within the first few weeks of Calculus Honors. **Show ALL of your work.** For all *Sketch* problems, ensure your answer is written on Graph Paper. Make note of difficult problems and questions you have as you work.

Please consult the sites under "Helpful Links" **on the last page of this assignment** to aid in your review. If you have any further questions as you work through the assignment, please feel free to email me. I will check my email throughout the summer and respond to any questions you have when I am able.

Bring this assignment with you on the **2nd** day of class. I look forward to seeing you in August.

Best,

Mr. Cantlin

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0.1 Algebra

1. Evaluate each expression without using a calculator

a) $(-3)^4$

b) -3^4

c) 3^{-4}

d) $\frac{5^{23}}{5^{21}}$

e) $(\frac{2}{3})^{-2}$

f) $16^{-\frac{3}{4}}$

2. Simplify each expression. Write your answer without negative exponents

a) $\sqrt{200} - \sqrt{32}$

b) $(3a^3b^3)(4ab^2)^2$

c) $(\frac{3x^{3/2}y^3}{x^2y^{-1/2}})^{-2}$

d) $\sqrt{32}\sqrt{2}$

3. Expand and simplify

a) $2x(x - 5)$

b) $3(x + 6) + 4(2x - 5)$

c) $(t - 5)^2 - 2(2 + 3)(8t - 1)$

d) $(4x - 1)(3x + 7)$

e) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$

f) $(1 + x - x^2)^2$

4. Factor each expression

a) $4x^2 - 25$

b) $x^2 - x - 6$

c) $2x^2 + 5x - 12$

d) $x^3y - 4xy$

e) $9x^2 - 36$

f) $5ab - 10abc$

5. Simplify the rational expression

a) $\frac{x^2+3x+2}{x^2-x-2}$

b) $\frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}}$

c) $\frac{1}{x+3} + \frac{1}{x^2-9}$

d) $\frac{x^2-1}{x^2-9x+8}$

e) $\frac{x^3+5x^2+6x}{x^2-x-12}$

6. Rationalize the expression and simplify

(a) $\frac{\sqrt{10}}{\sqrt{5}-2}$

(b) $\frac{\sqrt{4+h}-2}{h}$

7. Rewrite by completing the square

(a) $x^2 + 3x + 1$

(b) $2x^2 - 12x + 11$

8. Solve each equation

(a) $3x^2 + 5x + 1 = 0$

(b) $\frac{2x}{x+1} = \frac{2x-1}{x}$

(c) $x + 5 = 14 - \frac{1}{2}x$

(d) $4x - 5 = -2x + 7$

(e) $2x^2 + 7x + 2 = 0$

9. Solve each inequality. Write your answer in interval notation

(a) $1 - x \leq 2$

(b) $4 - 3x \geq 6$

(c) $x(x - 1)(x + 2) > 0$

(d) $-4 < 5 - 3x \leq 17$

(e) $|x - 4| < 3$

(f) $\frac{1}{x} < 4$

10. State whether each equation is true for all values of the variable(s). **Explain your reasoning**

(a) $\sqrt{ab} = \sqrt{a}\sqrt{b}$

(b) $\sqrt{x^2} = x$

(c) $(x^3)^4 = x^7$

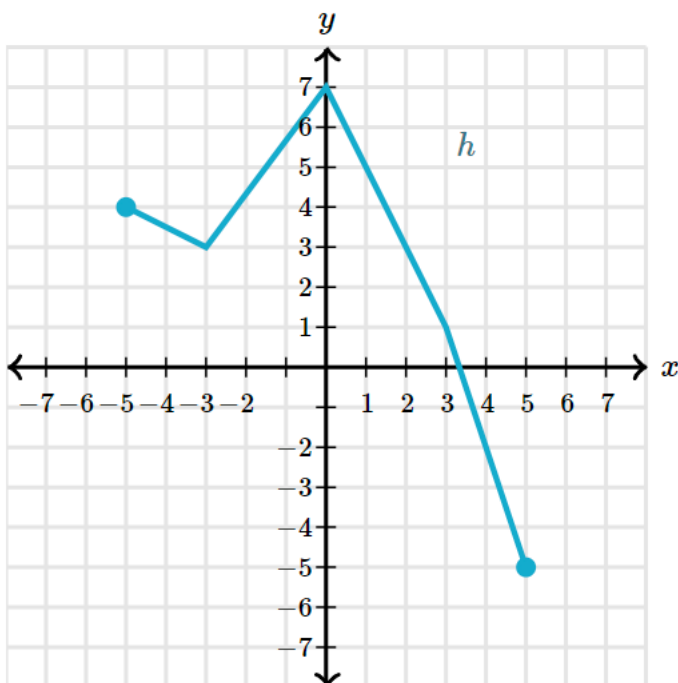
(d) $\sqrt{a^2 + b^2} = a + b$

(e) $6 - 4(x + a) = 6 - 4x - 4a$

0.2 Geometry

1. Find an equation for the line that passes through the point $(-1, 2)$ and
 - a) has slope 3
 - b) is parallel to the x-axis
 - c) is parallel to the y-axis
 - d) is parallel to the line $2x - 4y = 3$
2. Find an equation for the circle that has center $(3, 5)$ and passes through the point $(7, 0)$
3. Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 10y + 9 = 0$
4. Let $A(-1, 2)$ and $B(6, -7)$ be points in the plane
 - a) Find the slope of the line that contains A and B .
 - b) Find an equation of the line that passes through A and B . What are the x and y intercepts?
 - c) Find the midpoint of the segment AB
 - d) Find the length of the segment AB
 - e) Write an equation for the line through $(-1, 2)$ perpendicular to the line you found in b)
5. Sketch the region in the xy -plane defined by the equations or inequalities
 - a) $x^2 + y^2 < 4$
 - b) $x^2 + y^2 = 1$
 - c) $|x| < 4$
 - d) $-1 \leq y \leq 3$

0.3 Functions



1. The graph of a function f is given above

- a) State the value of $f(-3)$
- b) Estimate the value of $f(1.5)$
- c) For what values of x is $f(x) = 1$?
- d) State the domain and range of f

2. If $f(x) = x^3$, evaluate the quotient $\frac{f(2+h)-f(2)}{h}$ and simplify your answer

3. Find the domain of the following functions

a) $f(x) = \frac{2x+1}{x^2+x-2}$

b) $g(x) = \frac{\sqrt{x}}{x^2+1}$

c) $h(x) = \sqrt{4-x} + \sqrt{x^2-1}$

4. How are the graphs of the functions obtained from the graph of f ?

a) $y = -f(x)$

b) $y = 2f(x) - 1$

c) $y = f(x - 3) + 2$

5. Make a rough sketch of the graph of the following functions

a) $y = x^3$

b) $y = 4 - x^2$

c) $y = \sqrt{x}$

d) $y = -2^x$

e) $y = (x - 2)^3 + 3$

f) $y = 1 + x^{-1}$

6.

$$\text{Let } f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$$

(a) Evaluate $f(-2)$ and $f(1)$

(b) Sketch the graph of f

7. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions

a) $f \circ g$

b) $g \circ f$

c) $g \circ g \circ g$

8. Given

a) $f(x) = 3x - 2$, find $f^{-1}(x)$

b) $g(x) = \sqrt{x - 2}$, find $g^{-1}(x)$

c) $h(x) = \frac{x+4}{2x-5}$, find $h^{-1}(x)$

0.4 Trigonometry

1. Convert from degrees to radians

a) 300°

b) -18°

2. Convert from radians to degrees

a) $\frac{5\pi}{6}$

b) $\frac{\pi}{2}$

c) $\frac{3\pi}{4}$

d) 2

3. Find the length of an arc of a circle with radius 12cm if the arc subtends a central angle of 30°

4. Find the exact values

a) $\tan\left(\frac{\pi}{3}\right)$

b) $\sin\left(\frac{7\pi}{6}\right)$

c) $\sec\left(\frac{5\pi}{3}\right)$

d) $\cos(0)$

e) $\sin\left(\frac{\pi}{4}\right)$

f) $\cos\left(\frac{3\pi}{2}\right)$

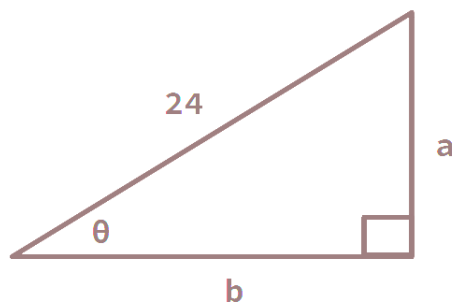
5. Prove the identities

a) $\tan \theta \sin \theta + \cos \theta = \sec \theta$

b) $\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$

6. Find all values of x such that $\sin 2x = \sin x$ and $0 \leq x \leq 2\pi$

7. Sketch the graph of the function $y = 1 + 2 \sin x$ without using a calculator



8. Express the lengths a and b in the figure above in terms of θ

Helpful Links

<https://www.khanacademy.org/math/algebra>

<https://www.khanacademy.org/math/geometry>

<https://www.khanacademy.org/math/algebra2>

<https://www.khanacademy.org/math/precalculus>

https://www.stewartcalculus.com/_update/media/common/reviewofalgebra.pdf

https://www.stewartcalculus.com/_update/media/common/reviewofanalgeom.pdf

<https://tutorial.math.lamar.edu/Classes/CalcI/ReviewIntro.aspx>

<https://www2.clarku.edu/faculty/djoyce/trig/identities.html>

<https://www.youtube.com/@patrickjmt/playlists>

<https://www.youtube.com/@TheOrganicChemistryTutor/playlists>

Note, while these links cover the material of the review, this list is **not** exhaustive. If you are unable to find material on content you need help with, please reach out to me and I would be happy to send you additional resources.